Junior Balkan MO 2006

1. If $n > 4$ is a composite number, then $2n$ divides $(n-1)!$.

2. The triangle ABC is isosceles with $AB = AC$, and $\angle BAC < 60^\circ$. The points D and E are chosen on the side AC such that, $EB = ED$, and $\angle ABD \equiv \angle CBE$. Denote by O the intersection point between the internal bisectors of the angles $\angle BDC$ and $\angle ACB$. Compute $\angle COD$.

3. We call a number *perfect* if the sum of its positive integer divisors(including 1 and n) equals $2n$. Determine all *perfect* numbers n for which $n-1$ and $n+1$ are prime numbers.

4. Consider a $2n \times 2n$ board. From the ith line we remove the central $2(i-1)$ unit squares. What is the maximal number of rectangles 2×1 and 1×2 that can be placed on the obtained figure without overlapping or getting outside the board?

