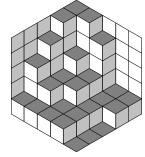




## Junior Balkan MO 2006



- 1 If  $n > 4$  is a composite number, then  $2n$  divides  $(n - 1)!$ .
- 2 The triangle  $ABC$  is isosceles with  $AB = AC$ , and  $\angle BAC < 60^\circ$ . The points  $D$  and  $E$  are chosen on the side  $AC$  such that,  $EB = ED$ , and  $\angle ABD \equiv \angle CBE$ . Denote by  $O$  the intersection point between the internal bisectors of the angles  $\angle BDC$  and  $\angle ACB$ . Compute  $\angle COD$ .
- 3 We call a number *perfect* if the sum of its positive integer divisors (including 1 and  $n$ ) equals  $2n$ . Determine all *perfect* numbers  $n$  for which  $n - 1$  and  $n + 1$  are prime numbers.
- 4 Consider a  $2n \times 2n$  board. From the  $i$ th line we remove the central  $2(i - 1)$  unit squares. What is the maximal number of rectangles  $2 \times 1$  and  $1 \times 2$  that can be placed on the obtained figure without overlapping or getting outside the board?